

# Quantum Zeno effect in atomic spin-exchange collisions

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The suppression of spin-exchange relaxation in dense alkali-metal vapors discovered in 1973 and governing modern atomic magnetometers is here reformulated in terms of quantum measurement theory and the quantum Zeno effect. This provides a new perspective of understanding decoherence in spin-polarized atomic vapors.

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## I. INTRODUCTION

Spin-exchange collisions [1], brought about by the Pauli exchange interaction, play a dominant role in the physics of spin-polarized atomic vapors and their applications [2]. Spin-exchange collisions are responsible not only for the very useful transfer of spin-polarization from one atomic species to another [3], but also for the detrimental effect they have on spin coherence, i.e. spin-exchange collisions cause decoherence [4]. The spin-coherence lifetime poses fundamental limitations to precision measurements involving spin-polarized atoms [5], as for example measurements of a small magnetic field (or a small Larmor frequency) performed with atomic magnetometers [6, 7, 8, 9]. However, it was early on realized [10, 11] that decoherence due to spin-exchange collisions can be suppressed if the spin-exchange rate is large enough relative to the frequency scale set by the atomic Larmor precession in an external magnetic field. In this work we will re-interpret this result in terms of quantum measurement theory [12]. In particular, we will reformulate this in terms of the quantum Zeno effect [13], the essence of which is that a frequent enough interrogation of a quantum system fundamentally alters its time evolution. We will also consider the physical information on the atomic spin state provided by these collision-induced measurements. From this perspective, we will also describe another kind of spin-dependent atomic collisions, namely spin-destruction collisions. The latter also lead to decoherence, which however is monotonically increasing with the collision rate, contrary to spin-exchange collisions. Whereas both kinds of collisions can be understood as performing a quantum measurement of the atomic spin coherence, they fundamentally differ on the route taken by the information provided by these measurements. In spin-exchange collisions, some information is in principle available, whereas in spin-destruction collisions the information is irretrievably lost in the environment. The reason for elaborating on this alternative perspective on spin-exchange collisions is that it motivated a recently discovered analogy to a seemingly dif-

ferent physical system, namely the charge-recombination of radical-ion-pairs [14].

## II. SPIN EXCHANGE COLLISIONS AS AN INFORMATION-RICH QUANTUM MEASUREMENT

In describing quantum measurements, we usually distinguish between the quantum system under consideration and the quantum probe which is an auxiliary quantum system. The probe interacts with the quantum system, all information on which is later extracted by performing measurements on the quantum probe [12]. The dissipative interaction of an open quantum system with its environment can also be molded into the previous picture, only now the probe system describing the environmental degrees of freedom is unobserved, i.e. information about the quantum system irretrievably leaks into the environment. While decoherence is present in both cases, in the former it is due to the unavoidable back-action of the probe onto the system, whereas in the latter due to information leakage to the environment.

In the specific case of  $N$  alkali-metal atoms confined in a cell, each atom is the quantum system, whereas all other atoms form a multitude of quantum probes. This distinction obviously fades away as we describe the combined system of  $N$  atoms, the behavior of which is an average over  $N$  separate quantum systems. The system degrees of freedom are embodied in the atomic spin state, described as usual [4] by the atom's  $2(2I+1)$ -dimensional ground state Hilbert space, where  $I$  is the atom's nuclear spin. The environmental degrees of freedom are found in the practically classical translational angular momentum of the atoms. The binary spin-exchange interaction Hamiltonian of two colliding atoms with electron spin  $\mathbf{s}_1$  and  $\mathbf{s}_2$  is of the form  $h_{se} = a(r)\mathbf{s}_1 \cdot \mathbf{s}_2$ , where  $a(r)$  is a function of the internuclear distance [2]. For one such collision we denote by  $\omega_{se}$  the integral of  $a(r)$  over the collision trajectory, hence the Hamiltonian describing one completed collision is  $\mathcal{H}_{se} = \omega_{se}\mathbf{s}_1 \cdot \mathbf{s}_2$  (in units  $\hbar = 1$ ). Obviously  $\omega_{se}$  depends on the particular collision trajectory. If  $\tau_c$  is the duration of the collision, then  $\phi_{se} = \omega_{se}\tau_c$  is the phase angle swept by each atomic spin during this collision. Due to the electrostatic nature

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of spin-exchange collisions [2],  $\phi_{se} \gg 1$ . By measuring  $\phi_{se}$  of atom 2 (the quantum probe), we can in principle extract information about the spin state of atom 1 (the quantum system). Indeed, the interaction Hamiltonian  $\mathcal{H}_{se}$  is interpreted by atom 2 as an effective magnetic field  $\mathbf{B} = \omega_{se}\langle\mathbf{s}_1\rangle$ , hence  $\phi_{se}$  is the precession angle of atom 2 spin in this magnetic field. Although extracting the value of  $\langle\mathbf{s}_1\rangle$  requires knowledge of the specific collision trajectory (hidden in the precise value of  $\omega_{se}$ ), the direction of  $\langle\mathbf{s}_1\rangle$  can be readily found from the sign of the phase rotation  $\phi_{se}$ . Another quantum probe (another atom) can extract similar directional information at a later time. A large number of such collisions is thus found to sample the atomic spin precession, hence from a series of such observations the spin-precession (Larmor) frequency  $\omega$  can be inferred. Needless to mention that this is not the way that  $\omega$  is measured in actual experiments. However, information being physical [15], the particular way of extracting it is inconsequential.

The uncertainty in such a measurement of  $\omega$  will be determined by the fact that the measurement cannot go on forever. Spin-exchange collisions will eventually produce a back-action on the measured quantum system. For small times and large spin-polarizations, this back-action is minimal [18]. However, as the spin-polarization decays, spin-exchange collisions will be able to induce a large phase jump  $\phi_{se}$  on the coherent spin precession of atom 1 (or, equivalently, any other atom). The number of such phase jumps per unit time will be given by the spin-exchange rate  $\gamma_{se} = nv\sigma_{se}$ , where  $n$  is the atom number density,  $v$  the mean relative velocity of two colliding atoms and  $\sigma_{se}$  the spin-exchange cross section. At long times, when the spin-polarization has decayed away, the measurement of the sign of  $\langle\mathbf{s}_1\rangle$  will merely reflect spontaneous spin noise [19, 20]. This collision-induced sampling process will thus result in a distribution of measured precession frequencies, the width of which will be on the order of  $\gamma_{se}$ . This is the spin-exchange broadening that limits the precision with which one can measure  $\omega$  [21]. From the view point of quantum measurements performed on an atom, the spin-exchange rate  $\gamma_{se}$  is identified with the measurement rate, i.e. the rate at which we extract information about the spin state of any given atom. An unexpected phenomenon is observed when  $\gamma_{se} \gg \omega$ : the width of the spin-resonance shrinks and scales as  $\omega^2/\gamma_{se} \ll \gamma_{se}$  [10, 11]. This is the quantum Zeno effect observed in the strongly interrogated atomic spin coherence. This dependence of the suppressed decoherence rate, i.e. the  $\omega^2/\gamma$  dependence is exactly the characteristic dependence of quantum Zeno effect, as has been described in [12] and more recently in [16]. It can be rephrased as follows: if the rate of performing measurements on (or extracting information from) a coherently evolving quantum system is larger than the system's evolution rate, the measurement-induced back-action on the system is suppressed.

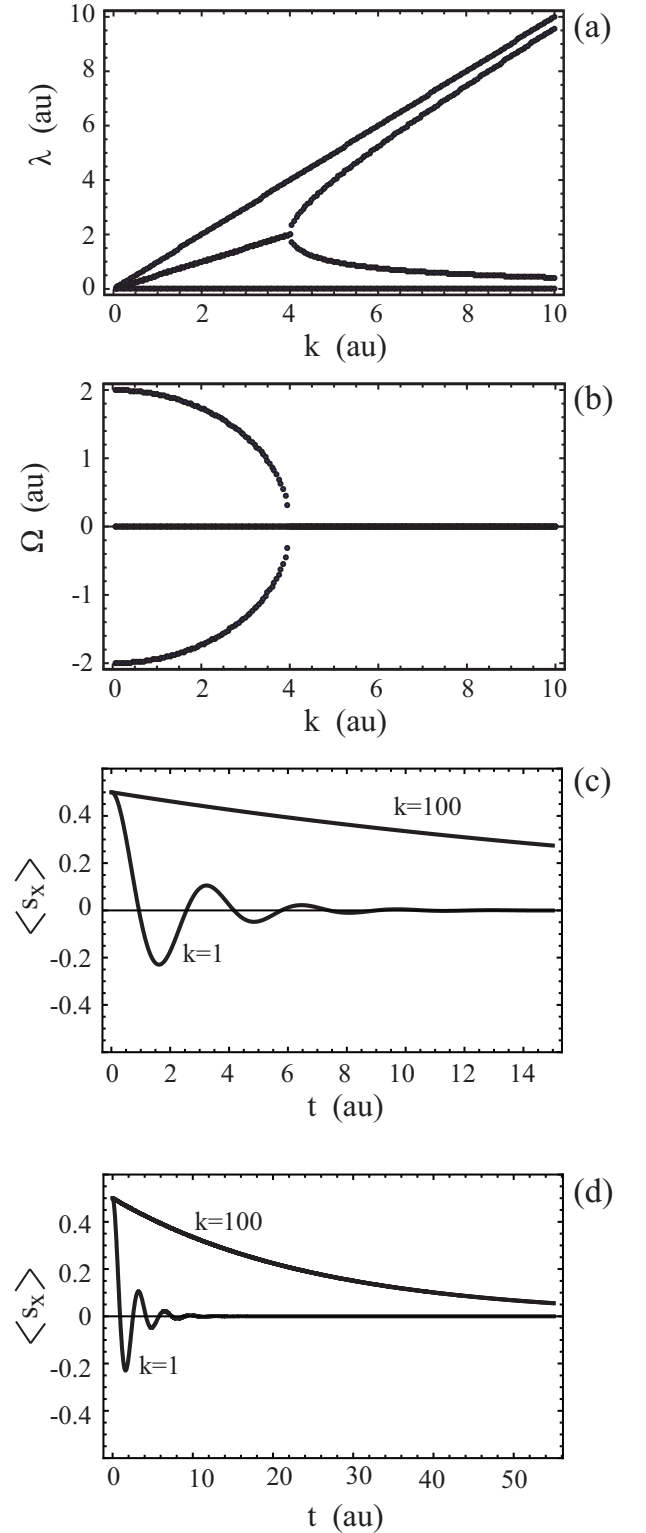


FIG. 1: Decay rates (a) and precession frequencies (b) corresponding to the eigenvalues of Eq. (1) for  $\omega = 1$ . (c) Time evolution of the expectation value  $\langle s_x \rangle$  for  $\omega = 2$  and two different values of the measurement rate  $k$ . The linear early-time dependence for  $k = 100$  is evident. (d) same as before, but for longer times.

### III. QUANTITATIVE ARGUMENTS

Towards a simplified quantitative argument, we describe the effects of collision-induced measurements on the atomic spin state by the density matrix equation

$$d\rho/dt = -i[H, \rho] - k[s_x, [s_x, \rho]] \quad (1)$$

where  $H = \omega s_z$  is the Zeeman interaction Hamiltonian, and the second (dissipative) term takes into account [12] the measurement of  $s_x$  at a rate  $k$ . In Fig. 1(a) and Fig. 1(b) we show the decay rates  $\lambda$  and precession frequencies  $\Omega$  of the four complex eigenvalues of (1), which are of the form  $-\lambda + i\Omega$ . The calculation was performed for constant  $\omega = 1$ . It is evident that one of the decay rates is suppressed when the measurement rate  $k \gg \omega$ . In Fig. 1(c,d) we show the time dependence of the expectation value  $\langle s_x \rangle$ , for two different values of the measurement rate  $k$ . It is seen that while at small values of  $k$  the coherent precession of  $\langle s_x \rangle$  decays at a rate proportional to  $k$ , at high measurement rates  $\langle s_x \rangle$  survives for a much longer time (in this simple model the precession frequency  $\Omega$  is also suppressed). In Fig. 1(c) in particular, the initial linear decay (in the case  $k = 100$ ) is clearly seen.

In reality, the effect of spin-exchange is described by a non-linear density matrix equation, that leads to similarly suppressed decay rates [11]. Moreover, spin-exchange collisions are different from the kind of measurements usually considered [17] in that they do not collapse the wavefunction to the initial state, but make atoms quantum-jump from one ground-state hyperfine-multiplet to the other. The Larmor spin precession has opposite sense in these two multiplets. However, the analog of the probability to find the system in the initial state which is usually considered in quantum Zeno effects [17] is in this case found in the correlation of the spin-coherence, i.e. the overlap between an unperturbed spin precession and one including such collision-induced jumps. Specifically, if we write  $\sigma(t) = \cos \omega t$  for the expectation value of the Pauli operator  $\sigma_x$ , and  $\sigma'(t)$  is the same function but including the occurrence of a jump in the precession frequency from  $\omega$  to  $-\omega$  at time  $\tau$ , then the average value of the correlation  $p =$

$(1/2\tau) \int_0^{2\tau} \sigma(t)\sigma'(t)dt$  can be approximated for  $\omega\tau \ll 1$  by

$$p \approx 1 - \left(\frac{\omega\tau}{2}\right)^2 \quad (2)$$

After  $N$  such independent collisions taking place in a total time interval  $T = N\tau$ , the overlap between the initial unperturbed precession and the one including  $N$  collisions will have decayed to

$$P = \left[1 - \left(\frac{\omega\tau}{2}\right)^2\right]^N \approx e^{-(\omega^2\tau/4)T} \quad (3)$$

Thus we recover the decay rate  $\omega^2/4\gamma_{se}$ , where  $\gamma_{se} = 1/\tau$  is the spin-exchange rate. This rather simplified analog of the rigorous statistical treatment [11] of spin-exchange collisions is meant to point out the dependence  $\omega^2/\gamma$  common to all appearances of the quantum Zeno effect in quantum systems characterized by an intrinsic frequency scale  $\omega$  and a measurement rate  $\gamma$ .

### IV. ABSENCE OF INFORMATION IN SPIN-DESTRUCTION COLLISIONS

Contrary to spin-exchange collisions which dissipate only spin coherence, there is another kind of binary collisions relaxing populations as well as coherences: the spin-destruction collisions [22], described by an interaction of the form  $\mathcal{H}_{sd} = 6\lambda(r)(\mathbf{s}_1 \cdot \hat{\mathbf{r}})(\mathbf{s}_2 \cdot \hat{\mathbf{r}}) - 2\lambda(r)\mathbf{s}_1 \cdot \mathbf{s}_2$ , where  $\hat{\mathbf{r}}$  is the unit vector along the internuclear axis, and  $\lambda(r)$  is a function of the internuclear distance. In the first term of  $\mathcal{H}_{sd}$  we have the direct participation of the environment degrees of freedom, i.e. it is this term that opens the loss-channel of spin-angular momentum into translational angular momentum. It is clear that not even the sign of  $\langle \mathbf{s}_1 \rangle$  can be inferred by observing the phase jump of atom 2 spin, since the effective magnetic field seen by atom 2 is now proportional to  $6(\langle \mathbf{s}_1 \rangle \cdot \hat{\mathbf{r}})\hat{\mathbf{r}} - 2\langle \mathbf{s}_1 \rangle$ . Since these two terms are of similar magnitude, the information loss into the environment is dominant no matter what the collision rate is.

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